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THE SEPARATION OF VISCOUS DRAG AND WAVE DRAG BY MEANS OF THE WAKE SURVEY

by

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NOTATION

D	Resistance or drag
g	The acceleration of gravity
H	A stagnation pressure, $k_2 + \rho gz$
H_o	A stagnation pressure, $k_1 + \rho gz$
I	A momentum transfer integral
k	Local stagnation pressure excluding the local hydrostatic head
P	A static pressure, $p_2 + \rho gz$
p	Local static pressure excluding the local hydrostatic pressure
Q	Source strength
S	Surface area
u_o	Free-stream velocity
u, v, w	Components in the direction of x-, y-, and z-axes, respectively, of local velocity
x, y, z	Three mutually perpendicular coordinates, Cartesian system
ρ	Density

<u>Subscripts</u>

A, A', G	Certain two-dimensional regions
E, T	Certain three-dimensional regions
B(E)	A potential-flow field in the region E
R(E)	A rotational-flow field in the region E
1	Conditions at box end A (Figure 1)
2	Conditions at box end A', real flow
3	Conditions at box end A', synthesized flow

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ABSTRACT

The wake-survey method of measuring viscous drag is analyzed for flows with surface waves and the result is obtained that the separation of wave and viscous drag is possible and that the method of separation is, without consideration of practical aspects, identical for submerged and surface flows.

INTRODUCTION

Certain important technical applications require knowledge of the total resistance and the magnitude of the components of the total resistance of bodies moving in real fluids. The total resistance may have contributions from various phenomena. For flows without surface phenomena (submerged flows) the resistance may be due to unsteady effects, viscous effects, "induced-flow" effects, and—for gas flows—the "supersonic wave" effects. The addition of surface effects (surface flows) causes an additional contribution.

The interest of the aerodynamicist has almost completely been in submerged flows. A desire to separate the contributions to the total resistance according to the nature of the contributing cause led Betz[1,2] to devise "A Method for the Direct Determination of Wing-Section Drag" for incompressible flows. The method utilizes pressure surveys in a plane normal to the flow direction and directly behind the body, the surveys being necessary only in the narrow wake region. The method, including the practical aspects, has been further discussed and justified,[3,4,5,6] and has become a standard technique of the aeronautical researcher. It has been used in both model and full-scale tests, and may be used for the investigation of generally practical configurations.

The desirability of separating the wave and viscous resistance associated with surface flows suggests the use of an appropriate wake-survey method for those flows. Although William Froude is usually credited with having first stated the principle upon which the method depends, the development of the method for surface flows has apparently never been accomplished;

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[1]References are listed on page 6.

it is not, at least, a standard naval-architecture research method. The derivation of the method follows. It makes use of the ideas of Betz, suitably modified according to the differences between submerged and surface flows.

The result is obtained that separation of wave and viscous drag is possible and that the method of separation is, without consideration of practical aspects, identical for submerged and surface flows.

DERIVATION

The resistance of a body in steady motion may always be found by a proper consideration of the conditions at the boundary of some space surface (a so-called momentum-control surface) completely containing the body. The body which creates surface phenomena is, for our purpose, enclosed within a rectangular box stationary with respect to the body, as in Figure 1. The end A of the box is taken at infinity ahead of the body, while end A' is taken at some close distance behind the body. The sides and bottom of the box are also taken an infinite distance from the body; the top need only be taken as being completely above the fluid surface.

The derivation does not require detailed knowledge of the surface phenomena, but it is assumed that only at considerable distances downstream

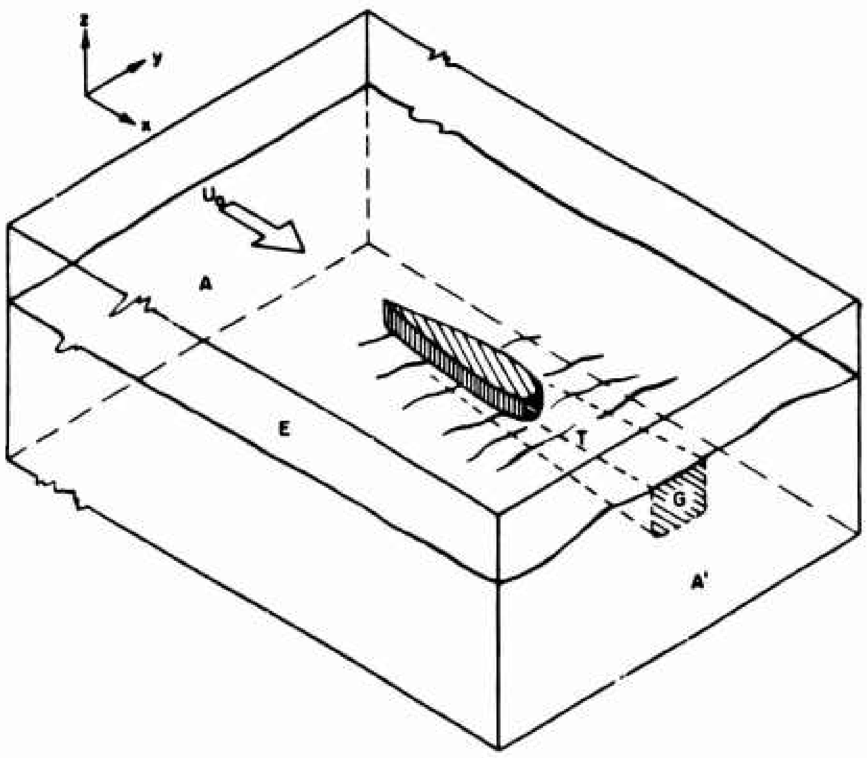

Figure 1 - The Momentum Control Surface About a Surface Flow

of the body does the surface phenomena involve significant rotational flow. Thus, inside and on the surface of the box (the region E) the flow is everywhere irrotational except in the finite region T which consists of the frictional wake. This region intersects the box end A' in the plane region G. The entire flow field may be considered to consist of the sum of a potential-flow field, B(E), and a rotational-flow field, R(E). The rotational field is null in the region E-T. The potential field within the region T is that continuation of the potential field in E-T which is consistent with the surface conditions in the region T; it may not be divergenceless.

The force on the body in the x-direction may be found by adding the net force in the x-direction "acting" on the control surface to the x-component of the net momentum transfer through the control surface.

Let:

The subscript 1 refer to conditions at box end A.

The subscript 2 refer to conditions at box end A' for the real flow.

The subscript 3 refer to conditions at box end A' for the synthesized flow.

p denote that part of the static pressure not due to hydrostatic forces.

u, v, w denote the velocity components in the x, y, z directions, respectively.

ρ denote the invariable fluid density.

u_0 denote the velocity of the stream at infinity upstream, it being assumed that the body is at rest in a moving stream.

Bernoulli's equation which applies to all parts of the fluid whose particle constituents have not passed through a region of vorticity is

$$\rho gz + p + \frac{1}{2}\rho(u^2 + v^2 + w^2) = k + \rho gz \qquad [1]$$

where the constant k may be referred to as the total head and g is the acceleration of gravity. All z's are, of course, referred to a certain common origin plane.

The expression for the resistance (or drag, D) becomes, for the case where a real surface flow is being considered

$$\begin{aligned} D &= \int_A\!\!\int (p_1+\rho gz+\rho u_1^2)\, dS - \int_{A'}\!\!\int (p_2+\rho gz+\rho u_2^2)\, dS + I \\ &= D_{viscous} + D_{wave} \end{aligned} \qquad [2]$$

where S represents a surface area and I represents the momentum transfer through the side and bottom surfaces. This transfer can be due only to the potential surface effects. It will be seen that it need not be explicitly given. The various integrals of Equation [2] are to be taken over infinite areas and permit no separation of the drag components. The further development depends upon a device for separating these components and obtaining an integral for the viscous drag which is to be taken over a finite region.

The derivation proceeds, using an extrapolation of Betz's idea, to consider the resistance due to a synthesized flow caused not by the real body, but by a fictitious wave source and a fictitious wake source. The appropriate wave source produces the divergenceless part of potential-flow field B(E) previously referred to. The velocity field corresponding to B(E) must necessarily correspond to the real velocity field in the region E-T, and it is assumed that the velocity field of the divergenceless part of B(E) within the region T may, because of continuity requirements, be considered identical with the real velocity field within T. The usefulness of the final result depends to a great extent upon the validity of this assumption and an assumption to be made concerning the existence of a wake source, but these assumptions should be as valid for surface flows as they are for submerged flows; the usefulness of the submerged flow result has been demonstrated.[5] Within T a static-pressure excess referred to the real flow exists due to the invariance of total head in the wave-source flow. In order to relieve the static-pressure excess a certain distribution of sources is utilized. It is assumed that there exists a wake-source distribution such that the surface wave pattern and the potential flow in region E-T is unaffected, but that in the region T increases in streamwise velocity components, and in only these components, are obtained just sufficient to remove the static pressure excess. The flow in the region A' then has the properties

$$k_3 = k_1; \quad p_3 = p_2; \quad v_3 = v_2; \quad w_3 = w_2; \quad u_3 > u_2 \qquad [3]$$

The resistance of the synthesized flow is the sum of the resistance of the wave source which is identical with D_{wave}, and the resistance of the fictitious wake source. This wake source of necessity has the strength

$$Q = \rho \iint_G (u_3 - u_2)\, dS \qquad [4]$$

and according to Lagally's Theorem[7] the wake source thus has a resistance

$$D_{wake\ source} = -\rho Q\, u_0 \qquad [5]$$

The results so far obtained:

$$D_{wave} + D_{viscous} = \iint_A (p_1+\rho gz+\rho u_1^2)\,dS - \iint_B (p_2+\rho gz+\rho u_2^2)\,dS + I \qquad [2]$$

$$D_{wave} + D_{wake\ source} = \iint_A (p_1+\rho gz+\rho u_1^2)\,dS - \iint_C (p_3+\rho gz+\rho u_3^2)\,dS + I \qquad [6]$$

$$D_{wake\ source} = -\rho u_0 \iint_G (u_3 - u_2)\,dS \qquad [7]$$

$$k_3 = k_1; \quad p_3 = p_2; \quad v_3 = v_2; \quad w_3 = w_2; \quad u_3 > u_2 \qquad [3]$$

Making use of these results and the result easily obtained from [1] and [3]

$$\rho(u_3^2 - u_2^2) = 2(k_1 - k_2) \qquad [8]$$

there is obtained the desired result

$$D_{viscous} = 2\iint_G (k_1 - k_2)\,dS - \rho u_0 \iint_G (u_3 - u_2)\,dS \qquad [9]$$

Betz obtained for submerged flows, in a slightly different manner, the result

$$D_{viscous} = \iint_G (k_1 - k_2)\,dS - \frac{\rho}{2} \iint_G (u_3 - u_2)(2u_0 - u_3 - u_2)\,dS \qquad [10]$$

Equations [9] and [10] are, however, identical since

$$\frac{\rho}{2} \iint_G (u_3 - u_2)(u_3 + u_2)\,dS = \frac{\rho}{2} \iint_G (u_3^2 - u_2^2)\,dS = \iint_G (k_1 - k_2)\,dS \qquad [11]$$

It only remains now to express Equation [9] in terms of measurable quantities. The simplest expression will be obtained if it is assumed that

$$\rho gz + p_2 + \rho\frac{u_2^2}{2} = k_2 + \rho gz \qquad [12]$$

that is, that the streamwise component of velocity behind the body is much greater than the other components.

Then, if $H_0 = k_1 + \rho gz$; $H = k_2 + \rho gz$; $P = \rho gz + p_2$, where H_0, H, and P are measurable, there results

$$\boxed{D_{viscous} = \iint \left\{ 2(H_0 - H) + (2\rho u_0^2[H - P])^{1/2} - (2\rho u_0^2[H_0 - P])^{1/2} \right\} dS} \qquad [13]$$

DISCUSSION

The theoretical possibility of separating the viscous and wave drags for surface flows by means of a wake survey has been demonstrated. The utility of the method depends upon the solution of those problems of practical nature which will arise in the application.

The separation of the viscous and wave drags in model tests would permit the possibility of improvement in drag prediction techniques through the replacement of the Froude method, in which the viscous drag is not measured, but is estimated by calculation.

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